

MODELLING DAILY VALUE-AT-RISK USING REALIZED VOLATILITY AND  
ARCH TYPE MODELS

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**Abstract**

In this paper, we compare the performance of a daily ARCH type model (which uses *daily* returns) with the performance of a model based on the daily realized volatility (which uses *intraday* returns) when the one-day ahead Value-at-Risk is to be computed. While the VaR specification based on a long memory skewed Student model for the daily realized volatility provides adequate one-day-ahead VaR forecasts for two stock indexes (the CAC40 and SP500) and two exchange rate returns (the YEN-USD and DEM-USD), it does not really improve on the performance of a VaR model based on the skewed Student APARCH model and estimated using daily data only. Thus both methods seem to be equivalent. This paper also shows that daily returns standardized by the square root of the one-day-ahead forecast of the daily realized volatility are not normally distributed.

*Keywords:* Value-at-Risk, realized volatility, skewed Student distribution, APARCH

*JEL classification:* C52, C53, G15

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# 1 Introduction

The recent widespread availability of databases providing the intraday prices of financial assets (stocks, stock indexes, bonds, currencies, . . . ) has led to new developments in applied econometrics and quantitative finance as far as the modelling of daily and intradaily volatility is concerned. Focusing solely on the modelling of daily volatility using intraday data, the recent literature suggests at least three possible methods for characterizing volatility and risk at an aggregated level, which we take to be equal to one day in this paper.

The first possibility is to sample the intraday data on a daily basis so that closing prices are recorded, from which daily returns are subsequently computed. In this setting, the notion of intraday price movements is not an issue, as the method is tantamount to estimating a volatility model on daily data. One well-known example is the ARCH model of Engle (1982) and subsequent ARCH type models such as the GARCH model of Bollerslev (1986) (see Palm, 1996, for a recent survey). The second method is based on the notion of realized volatility which was recently introduced in the literature by Taylor and Xu (1997) and Andersen and Bollerslev (1998) and which is grounded in the framework of continuous time finance with the notion of quadratic variation of a martingale. In this case, a daily measure of volatility is computed as an aggregated measure of volatility defined on intraday returns. More specifically, the daily realized volatility is computed as the sum of the squared intraday returns for the given trading day. We thus make explicit use of the intraday returns to compute the realized volatility, from which the daily volatility is modelled. This approach has been popularized recently in a collection of papers such as Andersen, Bollerslev, Diebold, and Labys (2000a), Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Diebold, and Labys (2001a) and Andersen, Bollerslev, Diebold, and Labys (2001b). Moreover this method has been shown to perform equally well for stocks and exchange rate returns. A third possibility is to estimate a high frequency duration model on price durations for the given asset, and then use this irregularly time-spaced volatility at the aggregated level. Examples are Engle and Russell (1997), Gerhard and Hautsch (2002) or Giot (2002).

In this paper we focus on the first two methods as our aggregation level is equal to one day, and it is not clear how duration models could be of any help in this situation. The recent literature on realized volatility (see the references given above) and the huge literature on daily volatility models seem to indicate that a researcher or market practitioner faces two distinct possibilities when daily volatility is to be modelled. Going one way or the other is however not a trivial question. If one decides to model daily volatility using daily realized volatility, then intraday data are needed so that corresponding intraday returns can be computed. Even today, intraday data remain relatively costly and are not readily available for all assets. Furthermore, a large amount of data handling and computer programming is usually needed to retrieve the needed intraday returns from the

raw data files supplied by the exchanges or data vendors. On the contrary, working with daily data is relatively simple and the data are broadly available. However, one has the feeling that all the relevant data are not taken into account, i.e. that by going at the intraday level one could get a much better model.

In this paper we aim to address this issue by comparing the performance of a daily ARCH type model (which uses *daily* returns) with the performance of a model based on the daily realized volatility (which uses *intradaily* returns) when the one-step ahead Value-at-Risk (VaR) measure is to be computed. This exercise is done for two stock indexes (the French CAC40 index and the American SP500 index) and two exchange rates vis-a-vis the US dollar (the YEN-USD and the DEM-USD) for which intraday data are available over a long time period (i.e. 5 years for the CAC40 and about 12 years for the other series). VaR modelling is a natural application of volatility models as VaR measures are directly related to the expected volatility over the relevant time horizon. VaR models have been developed since the middle of the 1990's to quantify and assess market risk in financial markets. Put simply, the goal of VaR models is to provide, at a given percentage level and for a portfolio of (marketable) assets, the most likely loss for a financial institution. For example, the VaR at level  $\alpha$  at the 1-day time horizon is the nominal 1-day loss that will not be exceeded in  $100 \cdot \alpha$  portfolio realizations out of 100. The literature on VaR models has grown remarkably over the last decade because of the popularity of the RiskMetrics VaR specification of JP Morgan (see below) and the risk-adjusted measures of capital adequacy enforced by the Basel committee. Textbook material is presented in Jorion (2000) or Saunders (2000) while recent applications of univariate time series models of the ARCH type to VaR problems are given in e.g. Lee and Saltoglu (2001), Danielsson (2002) or Berkowitz and O'Brien (2002). Because volatility is a key input to VaR models, the characterization of asset volatility (along with the density distribution of the asset returns) is of paramount importance when implementing and testing VaR models. We thus have daily and intradaily data for the stock indexes and exchange rates and aim to ascertain which data is the most useful to compute daily VaR measures. Indeed, (a) because we have intraday data over a long time period, we can retrieve the daily closing prices for the indexes and then compute daily VaR measure using daily ARCH type models; (b) with the intradaily data we can compute intraday returns and daily realized volatility, and we then have a competing model for the daily VaR which uses the intraday information.

Our main results can be summarized in one sentence: yes, an (adequate) ARCH type model can deliver accurate VaR forecasts and this model performs as well as a competing VaR model based on the realized volatility. The key issue is to use a daily ARCH type model that clearly recognizes and fully takes into account the key features of the empirical data such as a high kurtosis and skewness in the observed returns. In this paper we use the skewed Student APARCH model (see for instance Lambert and Laurent, 2001 and Giot and Laurent, 2001), which delivers excellent results when

applied to daily data. It is also true that the model based on the realized volatility delivers equally adequate VaR forecasts but this comes at the expense of using intraday information.<sup>1</sup> Thus, for the two indexes and exchange rates under review, the results clearly indicate that modelling the realized volatility may be useful, but it is far from being the only game in town. Moreover, we also extend previous results of Andersen, Bollerslev, Diebold, and Ebens (2001) by showing that daily returns standardized by the square root of the one-day-ahead *forecast* of the daily realized volatility are not normally distributed. This implies that, in a forecasting framework such as Value-at-Risk, realized volatility measures must be combined with adequate density distributions.

The rest of the paper is organized in the following way. In Section 2, we describe the available intraday data and characterize the stylized facts of the realized volatility. In Section 3, we introduce the two competing models (i.e. the skewed Student APARCH model for the daily returns and the model based on the realized volatility) and detail the estimation results. These two models are used to provide one-day-ahead VaR forecasts for the stock indexes and exchange rates in Section 4. Finally, Section 5 concludes.

## 2 Data and stylized facts

### 2.1 Data

The data are available for two stock indexes and two exchange rates on an intraday basis and for a relatively long period of time (see details below), which allows VaR modelling and testing over an extended period of time. For these four series we consider first daily returns (which are used by the skewed Student APARCH model) and then 15-minute and a 1-hour intraday returns for the stock indexes and the exchange rates respectively (these intraday returns are used to compute the daily realized volatility).

Our first asset is the French CAC40 stock index for the 1995-1999 time period, i.e. 1,249 daily observations. The CAC40 index is computed by the Paris Bourse (EURONEXT) as a weighted measure of the prices of its components. It is available in the database on an intraday basis with the index being computed approximately every 30 seconds. For the time period considered in this paper, the opening hours of the French stock market were from 10 am to 5 pm, thus 7 hours of trading per day. With a regularly time-spaced 15-minute grid, this translates into 28 intraday returns which are used to compute the daily realized volatility (see below for details). Intraday prices for the CAC40 index are computed every 15 minutes using a linear interpolation between the closest recorded prices below and above the time set in the grid. Correspondingly, all returns are computed as the first difference in the regularly time-spaced log prices of the index. Because

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<sup>1</sup>See also Martens (2001) for a comparative study of daily vs. realized volatility in a volatility forecasting framework, albeit the author does not focus on the density distributions of the returns.

the exchange is closed from 5 pm to 10 am of the next day, the first intraday return (computed at 10h15 when working with a 15-minute time grid for example) is the first difference between the log price at 10h15 and the log price at 5 pm the day before. Daily returns in percentage are defined as 100 times the first difference of the log of the closing prices.<sup>2</sup>

Our second dataset contains 12 years (from January 1989 to December 2000, or 3,241 daily observations) of tick-by-tick prices for the SP500 futures contracts traded on the Chicago Mercantile Exchange. Such SP500 futures contracts are traded from 8h30 to 15h10 Chicago time, i.e. from 9h30 to 16h10 New York time. To conveniently define 15-minute returns, we remove all prices recorded after 16h New York time.<sup>3</sup> Like for the CAC40 dataset, intraday prices at the 15-minute frequency are the outcomes of a linear interpolation between the closest recorded prices (for the nearest contract to maturity) below and above the times in the 15-minute grid.<sup>4</sup> Returns are computed as the first difference of the regularly time-spaced log prices of the index, with the overnight return included in the first intraday return. Daily returns in percentage are defined as 100 times the first difference of the log of the closing prices.

Our third and fourth datasets contain hourly data for two major exchange rates, the Japanese yen (YEN) and the Deutsche Mark (DEM) against the US Dollar (USD). For these two exchange rates, we have about 12 years of intraday data, from January 1989 to February 2001. During this period, the US Dollar, the DEM and the YEN were the main currencies traded in the FOREX market, and thus the volatility of the DEM-USD and YEN-USD currency returns made up most of the currency risk faced by large institutional investors and international corporations. The raw data consists of all interbank DEM-USD and YEN-USD bid-ask quotes displayed on the Reuters FAFX screen during this period. These quotes are indicative quotes but have been shown to be adequate inputs for computing intraday returns, see for example Danielsson and Payne (2002). Note that intraday FOREX returns computed from quoted bid-ask prices are subject to various market microstructure ‘frictions’, e.g. strategic quote positioning and inventory control. Such features are generally immaterial when analyzing longer horizon returns, but may distort the statistical properties of the underlying ‘fundamental’ high-frequency intraday returns. The sampling frequency at which such considerations become a concern is intimately related to market activity. For our exchange rate series, preliminary analysis based on the methods of Andersen, Bollerslev, Diebold, and Labys (2000b) and Oomen (2001) suggest that the use of equally-spaced thirty-minute or hourly returns strikes a satisfactory balance between the accuracy of the continuous-record asymptotics underlying the construction of our realized volatility measures

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<sup>2</sup>By definition and using the properties of the log distribution, the sum of the intraday returns is equal to the observed daily return based on the closing prices.

<sup>3</sup>Thus the last recorded price for the futures at 16h corresponds more or less to the closing price of the “cash” SP500 index computed from its constituents traded on the NYSE or NASDAQ.

<sup>4</sup>The choice of the nearest contract to maturity means that we always select very liquid futures contracts.

on the one hand, and the confounding influences from the market microstructure frictions on the other. As standard in the literature, we compute hourly exchange rate prices from the linearly interpolated logarithmic average of the bid and ask quotes for the two ticks immediately before and after the hourly time stamps throughout the global 24-hour trading day. Next we obtain hourly returns as 100 times the first difference of the equally time-spaced logarithmic prices. To avoid the so-called week-end effect (there are almost no currency trades during the week-end), we exclude all returns from Friday 21:00 GMT until Sunday 21:00 GMT.

## 2.2 Realized volatility: definition and stylized facts

Estimating and forecasting volatility is a key issue in empirical finance. After the introduction of the ARCH model by Engle (1982) or the Stochastic Volatility (SV) model (see Taylor, 1994) and their various extensions, a new generation of conditional volatility models has been advocated recently by Taylor and Xu (1997) and Andersen and Bollerslev (1998), i.e. models making use of the realized volatility. See also the more recent work by Andersen, Bollerslev, Diebold, and Ebens (2001) or Andersen, Bollerslev, Diebold, and Labys (2001a).

The origin of ‘realized volatility’ is not as recent as it would seem at first sight. Merton (1980) already mentioned that, provided data sampled at a high frequency are available, the sum of squared realizations can be used to estimate the variance of an *i.i.d.* random variable. Taylor and Xu (1997) and Andersen and Bollerslev (1998) (among others) show that daily realized volatility may be constructed simply by summing up intraday squared returns. Assuming that a day can be divided in  $N$  equidistant periods and if  $y_{i,t}$  denotes the intradaily return of the  $i^{th}$  interval of day  $t$ , it follows that the daily volatility for day  $t$  can be written as:

$$\left[ \sum_{i=1}^N y_{i,t} \right]^2 = \sum_{i=1}^N y_{i,t}^2 + 2 \sum_{i=1}^N \sum_{j=i+1}^N y_{j,t} y_{j-i,t}. \quad (1)$$

If the returns have a zero mean and are uncorrelated,  $\sum_{i=1}^N y_{i,t}^2$  is a consistent (see Andersen, Bollerslev, Diebold, and Labys, 2001) and unbiased<sup>5</sup> estimator of the daily variance  $\sigma_t^2$ . Because all squared returns on the right side of this equation are observed when intraday data (at equidistant periods) are available,  $\left[ \sum_{i=1}^N y_{i,t} \right]^2$  is called the daily *realized* volatility.

By summing high-frequency squared returns we may then obtain an ‘error free/model free’ measure of the daily volatility.<sup>6</sup> However, choosing a very high sampling frequency (e.g. 30-second

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<sup>5</sup>Areal and Taylor (2000) show that even if this estimator is consistent and unbiased, it has not the least variance when  $N$  is finite. These authors propose to weight the intraday squared returns by a factor proportional to the intraday activity. This deflator may be obtained easily by applying Taylor and Xu’s (1997) variance multiplier or the Flexible Fourier Function (FFF) of Andersen and Bollerslev (1997). Due to the strong similarity of the results with the ‘non weighted squared returns’, we do not report the results using Areal and Taylor’s (2000) approach.

<sup>6</sup>The theory of quadratic variation shows that, under suitable conditions, realized volatility is not only an

or 1-minute frequency) may introduce a bias in the variance estimate due to market microstructure effects (bid-ask bounces, price discreteness or non-synchronous trading). As a trade off between these two biases, Andersen, Bollerslev, Diebold, and Labys (2001a) propose the use of 5-minute returns to compute daily realized volatility. Using the FTSE-100 stock index on the 1990-2000 time period, Oomen (2001) shows that the realized volatility measure increases when the sampling interval decreases while the summation of the cross terms in Equation (1) decreases. By comparing the average daily realized volatility and the autocovariance bias factor, Oomen (2001) argues that the optimal sampling frequency for his dataset suggests using 25-minute returns. For our two stock indexes, a sampling frequency of about 15-minute was found to be optimal.<sup>7</sup> For the exchange rates, we choose a sampling frequency of one hour.

Although the empirical work on realized volatility is still in its infancy, some stylized facts have already been ascertained:<sup>8</sup>

- First, the unconditional distribution of the realized volatility is highly skewed and kurtosed. On the other hand, the unconditional distribution of the logarithmic realized volatility is nearly Gaussian.
- Secondly, the (logarithmic) realized volatility appears to be fractionally integrated. Indeed, volatility shocks die out very slowly, neither in accordance with an ARMA structure (which implies an exponential decay) nor with a unit root process. Correspondingly, the ACF decreases very slowly.
- Finally, according to Ebens (1999) who analyzes the Dow Jones Industrial portfolio over the January 1993 to May 1998 period, the (logarithmic) realized volatility of stock indexes are non-linear in returns. This feature is also well known in ARCH type models and is known as the leverage effect: past negative shocks on the stock index returns have a larger impact on current (realized) volatility than past positive shocks (see Black, 1976; French, Schwert, and Stambaugh, 1987; Pagan and Schwert, 1990 and Zakoian, 1994). Note that an alternative explanation for this asymmetry is provided by the ‘volatility feedback’ effect, see Campbell and Hentschel (1992). This also holds for realized stock volatility, but the leverage effect seems to be of minor economic importance, see Andersen, Bollerslev, Diebold, and Ebens (2001).

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unbiased ex-post estimator of daily return volatility, but also asymptotically free of measurement error (Andersen, Bollerslev, Diebold, and Labys, 2001).

<sup>7</sup>To find the optimal sampling frequency, Oomen (2001) suggests to plot both the sum of squared intra-daily returns and the autocovariance bias factor versus the sampling frequency. The ‘optimal’ sampling frequency is chosen as the highest available frequency for which the autocovariance bias term has disappeared.

<sup>8</sup>In a preliminary version we highlighted these stylized facts on the two stock indexes. See Giot and Laurent (2001a) for more details. Additional stylized facts about realized volatility are available in Andersen, Bollerslev, Diebold, and Ebens (2001) or Andersen, Bollerslev, Diebold, and Labys (2001a).

### 3 Two competing models

Realized volatility was reviewed in the preceding section and we can now introduce an econometric model for the daily VaR based on this measure. Subsection 3.2 is devoted to this topic. As the goal of the paper is to compare the performance of an ARCH type model directly applied to the daily data with the performance of a model based on the realized volatility, we also need to characterize the skewed Student APARCH model that will be needed for the daily data. This is done in the next subsection.

#### 3.1 ARCH type models

A series of asset returns  $y_t$  ( $t = 1, \dots, T$ ), known to be conditionally heteroscedastic, is typically modelled as follows:

$$y_t = \mu_t + \varepsilon_t \tag{2}$$

$$\varepsilon_t = \sigma_t z_t \tag{3}$$

$$\mu_t = c(\eta|\Omega_{t-1}) \tag{4}$$

$$\sigma_t = h(\eta|\Omega_{t-1}), \tag{5}$$

where  $c(\cdot|\Omega_{t-1})$  and  $h(\cdot|\Omega_{t-1})$  are functions of  $\Omega_{t-1}$  (the information set at time  $t-1$ ), and depend on an unknown vector of parameters  $\eta$ ;  $z_t$  is an independently and identically distributed (*i.i.d.*) process, independent of  $\Omega_{t-1}$ , with  $E(z_t) = 0$  and  $Var(z_t) = 1$ ;  $\mu_t$  is the conditional mean of  $y_t$  and  $\sigma_t^2$  is its conditional variance.

Because daily returns are known to exhibit some serial autocorrelation, we fit an AR( $n$ ) structure on the  $y_t$  series for all specifications:<sup>9</sup>

$$\Psi(L)(y_t - \mu) = \varepsilon_t, \tag{6}$$

where  $\Psi(L) = 1 - \psi_1 L - \dots - \psi_n L^n$ . In this case,  $\mu_t = \mu + \sum_{i=1}^n \psi_i (y_{t-i} - \mu)$ .

We now consider two possible specifications for the conditional variance of  $\varepsilon_t$ . First we detail one of the most simple (but widely used by market practitioners) measure of volatility, the RiskMetrics volatility specification. Next we consider the APARCH model, which is one of the most flexible ARCH-type model.

##### 3.1.1 RiskMetrics

In its most simple form, it can be shown that the basic RiskMetrics model is equivalent to a normal Integrated GARCH (1, 1) model where the autoregressive parameter is set at a prespecified value

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<sup>9</sup>The serial autocorrelation found in daily returns is not necessarily at odds with the efficient market hypothesis. See Campbell, Lo, and MacKinlay (1997) for a detailed discussion.

of 0.94 and the coefficient of  $\varepsilon_{t-1}^2$  is equal to 0.06. In this specification, we have that  $z_t$  is *i.i.d.*  $N(0, 1)$  and  $\sigma_t^2$  is defined as:

$$\sigma_t^2 = 0.06\varepsilon_{t-1}^2 + 0.94\sigma_{t-1}^2. \quad (7)$$

Therefore the RiskMetrics specification does not require estimation of unknown parameters in the volatility equation as all parameters are preset at given values. Although this is a crude way of modelling volatility, it is widely used by practitioners as this method often gives acceptable short-term volatility forecasts and it is very simple to use/program in Excel sheets.

### 3.1.2 Skewed Student APARCH

Recently, Giot and Laurent (2001b) have shown that (unlike the RiskMetrics model or more simple ARCH-type models) the skewed Student APARCH (in short SKST APARCH) model does provide accurate VaR forecasts, both for the right and left tails of the distribution of returns. The APARCH (Ding, Granger, and Engle, 1993) is an extension of the GARCH model of Bollerslev (1986) that nests at least seven GARCH specifications. In the case of an APARCH(1,1), Equation (5) is:

$$\sigma_t = \left[ \omega + \alpha_1 (|\varepsilon_{t-1}| - \alpha_n \varepsilon_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta \right]^{1/\delta}, \quad (8)$$

where  $\omega, \alpha_1, \alpha_n, \beta_1$  and  $\delta$  are additional parameters to be estimated.  $\delta$  ( $\delta > 0$ ) plays the role of a Box-Cox transformation of  $\sigma_t$ , while  $\alpha_n$  ( $-1 < \alpha_n < 1$ ), reflects the so-called leverage effect. The properties of the APARCH model have been studied recently by He and Teräsvirta (1999a, 1999b).

In VaR applications, the choice of an appropriate distribution for the innovation process ( $z_t$ ) is an important issue as it directly affects the ‘quality’ of the estimation of the required quantiles. As in Giot and Laurent (2001b) or Giot (2003), we use a standardized version of the skewed Student distribution introduced by Fernández and Steel (1998).<sup>10</sup> According to Lambert and Laurent (2001) and provided that  $v > 2$ , the innovation process  $z_t$  is said to be (standardized) skewed Student distributed, i.e.  $z_t \sim SKST(0, 1, \xi, v)$ , if:

$$f(z_t|\xi, v) = \begin{cases} \frac{2}{\xi + \frac{1}{\xi}} sg[\xi(sz_t + m)|v] & \text{if } z_t < -\frac{m}{s} \\ \frac{2}{\xi + \frac{1}{\xi}} sg[(sz_t + m)/\xi|v] & \text{if } z_t \geq -\frac{m}{s} \end{cases}, \quad (9)$$

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<sup>10</sup>Giot and Laurent (2001b) show that an AR-APARCH model with a skewed Student density succeeds in correctly forecasting (both in- and out-of-sample) the one-day-ahead VaR for three international stock indexes and three U.S. stocks of the Dow Jones index. Models based on the normal or Student distributions clearly underperform when applied to the same datasets.

where  $g(\cdot|v)$  is a symmetric (unit variance) Student density and  $\xi$  is the asymmetry coefficient.<sup>11</sup> Parameters  $m$  and  $s^2$  are respectively the mean and the variance of the non-standardized skewed Student:

$$m = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \quad (10)$$

and

$$s^2 = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2. \quad (11)$$

In short,  $\xi$  models the asymmetry, while  $v$  accounts for the tail thickness. See Lambert and Laurent (2001) for a discussion of the link between these two parameters and the skewness and the kurtosis.

### 3.2 Realized volatility model

Regarding the realized volatility, its main features are that the logarithmic realized volatility is approximately normal, appears to be fractionally integrated and correlated with past negative shocks (see Section 2.2). To take these properties into account, let us consider the following ARFIMAX(0, $d$ ,1) model (initially developed by Granger, 1980 and Granger and Joyeux, 1980 among others):

$$(1-L)^d(\ln RV_t - \mu_0 - \mu_1 y_{t-1} - \mu_2 y_{t-1}^-) = (1 + \theta_1 L)u_t \quad (12)$$

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} L^k,$$

where  $L$  is the lag operator,  $\mu_0$ ,  $\mu_1$ ,  $\mu_2$ ,  $\theta_1$  and  $d$  are parameters to be estimated,  $u_t$  is an *i.i.d.* random process with mean 0 and variance  $\sigma_u$ ,  $\ln RV_t$  is the logarithm of the realized volatility computed from the intraday returns observed for day  $t$ ,  $y_t$  is the daily return on day  $t$ ,  $y_t^-$  takes the value 0 when  $y_t > 0$  and the value  $y_t$  when  $y_t < 0$ . Note that to determine the orders of this ARFIMA model we rely on the Schwarz (SC) information criterion.

When  $u_t \sim N(0, \sigma_u^2)$ , we have by definition that  $\exp(u_t) \sim \log N(0, \sigma_u^2)$  (where  $\log N$  denotes the log-normal distribution). Thus, the conditional realized volatility is computed according to:

$$RV_{t|t-1} = \exp\left(\ln RV_t - \hat{u}_t + \frac{1}{2}\hat{\sigma}_u^2\right), \quad (13)$$

where  $\hat{u}_t$  denotes the estimated value of  $u_t$  by Equation (12) and  $\hat{\sigma}_u^2$  is the estimated variance of  $u_t$  in the same equation.

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<sup>11</sup>The asymmetry coefficient  $\xi > 0$  is defined such that the ratio of probability masses above and below the mean is  $\frac{\Pr(\epsilon \geq 0|\xi)}{\Pr(\epsilon < 0|\xi)} = \xi^2$ . Note also that the density  $f(\epsilon|1/\xi, v)$  is the symmetric of  $f(\epsilon|\xi, v)$  with respect to the mean. Therefore, working with  $\ln(\xi)$  might be preferable to indicate the sign of the skewness.

To compute a one-day-ahead forecast for the VaR of the daily returns  $y_t$  using the conditional realized volatility, we now reestimate the model defined by Equations (2)-(5), where the conditional mean is the usual AR( $n$ ) process while the conditional variance is proportional to  $RV_{t|t-1}$  detailed above, i.e.  $\sigma_t^2 = \sigma^2 RV_{t|t-1}$  (with  $\sigma^2$  being an additional parameter to be estimated). In other words all the dynamics of the conditional variance is assumed to be captured by the ARFIMAX model. This assumption will be tested in the empirical application. Note that  $\sigma^2$  is used to ensure that the rescaled innovations of the second step have a unit variance.

This specification is almost identical to the specification introduced in Subsection 3.1, but in this case the conditional volatility for the daily returns is proportional to the conditional realized volatility  $RV_{t|t-1}$  and thus uses the information provided by the intraday returns of day  $t-1$ . As in Subsection 3.1, an adequate distribution for the innovation process ( $z_t$ ) should be selected. The recent empirical literature (see Andersen, Bollerslev, Diebold, and Ebens, 2001) has stressed that daily returns standardized by their ex-post realized volatility ( $RV_t$ ) are nearly Gaussian. Hence an obvious first choice for the distribution of  $z_t$  would be the Gaussian distribution. However, because we want to forecast the one-day-ahead VaR,  $RV_t$  is not observed at time  $t-1$  and one has to rely on its one-day-ahead forecast, i.e.  $RV_{t|t-1}$ . As shown in the next subsection, this seemingly minor change has far-reaching consequences as it invalidates the choice of the normal distribution as an adequate distribution for  $z_t$ . Therefore, we suggest the use of the skewed Student distribution introduced above (or SKST RV model) to take into account the skewness and kurtosis of  $z_t$ . We also present results for the normal distribution (in short Normal RV model) to motivate our choice of a more flexible density distribution and allow for a comparison with recent results given in the literature on realized volatility.

### 3.3 Estimation results

In this subsection, we report estimation results for the two competing models detailed in Sections 3.1 and 3.2. Estimation of the GARCH-type models is done by approximate maximum likelihood using G@RCH 3.0 (see Laurent and Peters, 2002). Estimation of Equation (12) is carried out by exact maximum likelihood (Sowell, 1992) under the normality assumption using ARFIMA 1.0 (see Ooms and Doornik, 1998 and Doornik and Ooms, 1999) and conditional sum-of-squares maximum likelihood (Hosking, 1981) using G@RCH 3.0 (see Laurent and Peters).<sup>12</sup> Because the estimation procedures provide very similar outcomes, we only report the results obtained with the first method. Finally, estimation of the second step involved in the realized volatility approach is carried out using G@RCH 3.0. The Ox codes are available upon request.

Our first framework uses the daily models defined in Sections 3.1.1 and 3.1.2. To simplify the

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<sup>12</sup>The finite sample properties of the conditional sum-of-squares maximum likelihood have been investigated by Chung and Baillie (1993).

layout of the tables we only report the results pertaining to the conditional variance equation and the skewed Student density. Note that, based on the SC criterion, an AR(1) specification has been selected for the conditional mean of the SP500 index returns while no dynamics is needed for the other series. To check the adequacy of the overall conditional density, we first rely on the Pearson goodness-of-fit test that compares the empirical distribution with the theoretical one (see the application). For a given  $g$  number of cells, the Pearson goodness-of-fit statistics is:

$$P(g) = \sum_{i=1}^g \frac{(n_i - En_i)^2}{En_i}, \quad (14)$$

where  $n_i$  is the number of observations in cell  $i$  and  $En_i$  is the expected number of observations (based on the ML estimates). For *i.i.d.* observations and under the null of a correct distribution,  $P(g)$  is distributed as a  $\chi^2(g-1)$ . Actually, as shown by Palm and Vlaar (1997), the asymptotic distribution of  $P(g)$  is bounded between a  $\chi^2(g-1)$  and a  $\chi^2(g-k-1)$  where  $k$  is the number of estimated parameters. Since our conclusions hold for both critical values and for various values of  $g$ , we report the significance levels relative to  $\chi^2(g-1)$ , with  $g = 20$ , i.e.  $P(20)$  in the tables.

The results given in Table 1 show that:

- $\beta_1$  is close to 1 but significantly different from 1 for all series, which indicates a high degree of volatility persistence.<sup>13</sup> Furthermore, in all cases the APARCH models are stationary in the sense that  $\alpha_1 E(|z| - \alpha_n z)^\delta + \beta_1$  is lower than 1.
- $\delta$  is close to 1 for the SP500 index and not significantly different from 2 for the other series: the APARCH models the conditional standard deviation for the SP500 and the conditional variance for the other time series.
- For the stock indexes,  $\alpha_n$  is significantly positive: negative returns lead to higher subsequent volatility than positive returns (asymmetry in the conditional variance). However, no leverage effect is detected for the exchange rates.
- $v$  is much larger for the CAC40 index than for the other series: daily returns of the SP500 index, YEN-USD and DEM-USD exchange rates display a much larger kurtosis and exhibit fatter tails than returns for the French data.
- For the two exchange rates,  $\ln(\xi)$  is significantly different from 0, which indicates that the innovations of the YEN-USD (resp. DEM-USD) returns are positively (resp. negatively) skewed while those of the CAC40 and the SP500 indexes are symmetric.

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<sup>13</sup>Tse (1998) extended the APARCH by including a pure long memory feature (FIAPARCH). LR tests between the APARCH and the FIAPARCH clearly reject the FIAPARCH specification.

Table 1: Skewed Student (SKST) APARCH

	CAC40	SP500	YEN-USD	DEM-USD
$\omega$	0.196 (0.011)	0.006 (0.003)	0.011 (0.004)	0.006 (0.003)
$\alpha_1$	0.040 (0.014)	0.054 (0.010)	0.076 (0.015)	0.048 (0.010)
$\alpha_n$	0.439 (0.189)	0.583 (0.108)	-0.039 (0.069)	0.026 (0.066)
$\beta_1$	0.944 (0.015)	0.953 (0.009)	0.916 (0.015)	0.944 (0.011)
$\delta$	1.783 (0.533)	0.980 (0.144)	1.712 (0.302)	1.840 (0.310)
$\ln(\xi)$	-0.062 (0.042)	-0.026 (0.024)	0.087 (0.025)	-0.059 (0.025)
$\nu$	13.254 (4.625)	5.542 (0.515)	5.243 (0.499)	6.757 (0.773)
$Q_{20}$	23.246 [0.277]	22.617 [0.255]	15.629 [0.739]	15.357 [0.756]
$Q_{20}^2$	13.824 [0.740]	14.924 [0.667]	18.111 [0.448]	7.810 [0.981]
$P_{20}$	13.282 [0.834]	10.543 [0.938]	13.847 [0.793]	14.840 [0.753]
$\alpha_1 E( z  - \alpha_n z)^\delta + \beta_1$	0.992	0.994	0.985	0.990

Estimation results for the volatility specification of the skewed Student APARCH model on the daily returns of the CAC40 and SP500 stock indexes and the YEN-USD and DEM-USD exchange rates. Standard errors are reported in parentheses.  $Q_{20}$  and  $Q_{20}^2$  are respectively the Ljung-Box Q-statistic of order 20 computed on the standardized residuals and squared standardized residuals.  $P_{20}$  is the Pearson goodness-of-fit statistics with 20 cells. P-values of the statistics are reported in brackets

- The AR-APARCH succeeds in taking into account all the dynamical structure exhibited by the returns and volatility of the returns as the Ljung-Box on the standardized residuals ( $Q_{20}$ ) and the squared standardized residuals ( $Q_{20}^2$ ) are always non-significant at the 5% level.
- Finally, the relevance of the skewed Student APARCH model is ascertained by the Pearson goodness-of-fit statistic  $P(20)$ .

In our second framework we explicitly use the information provided by the intradaily returns to compute the daily realized volatility. We first estimate an ARFIMAX(0, $d$ ,1) model on the logarithmic realized volatility  $\ln RV_t$  as in Equation (12). In a second step, we assume that the conditional mean of  $y_t$  follows the same AR structure as previously, while the conditional variance of the daily returns is expressed as  $\sigma_t^2 = \sigma^2 RV_{t|t-1}$ , with  $\sigma^2$  being an additional parameter to be estimated. Note that to perform this second step, one has to make an additional assumption on the innovation process. As outlined at the end of Section 3.2, one assumes first that  $z_t$  is normally distributed and then skewed Student distributed. Table 2 presents estimation results for this second method. Panel I of the table deals with the ARFIMAX specification while Panel II is related to the second step. Note that to save space, we do not report the estimated parameters of the conditional mean in Panel II.

Several comments can be made:

- First, the specification seems to succeed in modelling the dynamics of the first two conditional moments of the series. Indeed, the Ljung-Box statistics  $Q_{20}$  and  $Q_{20}^2$  indicate that the serial correlation in the error term and its square has been taken care of (at the conventional levels of significance).
- Parameter  $d$  is well above 0 for the four series. Actually, one has the typical value of 0.4 for the exchange rates (see Andersen, Bollerslev, Diebold, and Labys, 2001) and a slightly larger value for the stock indexes. In all cases,  $d$  is lower than 0.5 (even if not statistically different from 0.5 for the stock indexes), which indicates that the logarithm of the realized volatility might be covariance-stationary.
- $\mu_2$  is significantly negative for all series: negative returns lead to higher subsequent volatility than positive returns (asymmetry in the conditional variance similar to the APARCH model). Note that this result differs from the one obtained with the APARCH model, where no leverage effect was detected for the exchange rates.
- While the recent literature has stressed that ex-post standardized returns have an almost normal distribution (see Andersen, Bollerslev, Diebold, and Labys, 2000 or Andersen, Bollerslev, Diebold, and Ebens, 2001), this is certainly not true for ex-ante standardized returns, i.e. returns standardized by the square root of the one-day-ahead *forecast* of the daily realized

Table 2: Realized volatility framework

	CAC40	SP500	YEN-USD	DEM-USD
Panel I				
$\mu_0$	-0.016 (0.729)	-0.565 (1.120)	-1.123 (0.465)	-1.079 (0.369)
$\mu_1$	0.029 (0.026)	-0.016 (0.020)	0.256 (0.030)	0.078 (0.032)
$\mu_2$	-0.187 (0.035)	-0.215 (0.034)	-0.386 (0.055)	-0.184 (0.032)
$\theta_1$	-0.341 (0.053)	-0.287 (0.030)	-0.218 (0.037)	-0.209 (0.037)
$d$	0.463 (0.035)	0.480 (0.019)	0.416 (0.027)	0.407 (0.027)
$\sigma_u^2$	0.444	0.399	0.615	0.484
Panel II				
$\sigma^2$	1.032 (0.046)	1.106 (0.041)	1.008 (0.043)	1.099 (0.039)
$\ln(\xi)$	-0.069 (0.041)	-0.018 (0.024)	0.079 (0.025)	-0.054 (0.025)
$v$	16.070 (6.658)	6.097 (0.621)	5.246 (0.501)	6.808 (0.788)
$Q_{20}$	24.664 [0.215]	23.095 [0.233]	15.195 [0.765]	14.802 [0.788]
$Q_{20}^2$	16.745 [0.669]	3.221 [0.999]	34.045 [0.026]	23.075 [0.285]
$P_{20}$	17.994 [0.523]	13.185 [0.829]	19.446 [0.428]	9.856 [0.956]

Panel I: Estimation results for the logarithm of the realized volatility (defined on 15-minute returns for the CAC40 and SP500 stock indexes and 1-hour returns for the exchange rates) using an ARFIMAX(0,d,1) specification:  $(1 - L)^d(\ln RV_t - \mu_0 - \mu_1 y_{t-1} - \mu_2 y_{t-1}^-) = (1 + \theta_1 L)u_t$ . Panel II:  $y_t = \mu_t + \sqrt{\sigma^2 RV_{t|t-1}} z_t$ , with  $z_t \sim SKST(0, 1, \xi, v)$ . Standard errors are reported in parentheses.  $Q_{20}$  and  $Q_{20}^2$  are respectively the Ljung-Box Q-statistic of order 20 computed on the standardized residuals and squared standardized residuals.  $P_{20}$  is the Pearson goodness-of-fit statistics with 20 cells. P-values of the statistics are reported in brackets.

volatility. The estimated parameters  $\ln(\xi)$  and  $v$  reported in Table 2 suggest that the ex-ante standardized returns of the CAC40, SP500, YEN-USD and DEM-USD are kurtosed and skewed (this is particularly true for the exchange rates).<sup>14</sup> Hence, while daily returns standardized by the ex-post square root of the daily realized volatility are normally distributed, the same daily returns standardized by the square root of the one-day-ahead *forecast* of the daily realized volatility are not normally distributed. As shown in the next section on Value-at-Risk, this implies that, in a forecasting framework, realized volatility measures must be combined with adequate density distributions.

## 4 Assessing the VaR performance of the models

In the last part of the paper, we now focus on the application of the volatility models detailed previously to a Value-at-Risk application. As indicated in the introduction where we briefly reviewed the VaR framework, our goal in this paper is to ascertain if volatility models based on the realized volatility improves on the one-day ahead daily VaR forecasts made by a model based on daily returns. In both cases, we choose the skewed Student density distribution as the density distribution for the standardized error term to take into account the salient features of the returns standardized by the volatility forecast. The latter is first equal to the daily volatility forecast (skewed Student APARCH model), and then based on the realized volatility measure (skewed Student realized volatility model). We also present results for the popular RiskMetrics method and the normal realized volatility model.

More specifically, we use the estimated coefficients reported in the previous tables to compute the volatility forecasts used as inputs in the computation of the one-day-ahead VaR for the four series. When assuming a normal distribution for the innovations, the VaR for long trading positions (i.e. left tail of the density distribution of returns) is given by  $\mu_t + z_\alpha \sigma_t$ , while for short trading positions (i.e. right tail of the density distribution of returns) it is equal to  $\mu_t + z_{1-\alpha} \sigma_t$ , with  $z_\alpha$  being the left quantile at  $\alpha\%$  for the normal distribution and  $z_{1-\alpha}$  the right quantile at  $\alpha\%$ . When assuming a skewed Student distribution, the VaR for long trading positions is given by  $\mu_t + skst_{\alpha,v,\xi} \sigma_t$  and  $\mu_t + skst_{1-\alpha,v,\xi} \sigma_t$ , with  $skst_{\alpha,v,\xi}$  being the left quantile at  $\alpha\%$  for the skewed Student distribution with  $v$  degrees of freedom and asymmetry coefficient  $\xi$ ;  $skst_{1-\alpha,v,\xi}$  is the corresponding right quantile.<sup>15</sup> If  $\ln(\xi)$  is smaller than zero (or  $\xi < 1$ ),  $|skst_{\alpha,v,\xi}| > |skst_{1-\alpha,v,\xi}|$  and the VaR for long trading positions will be larger (for the same conditional variance) than the VaR for short trading positions. When  $\ln(\xi)$  is positive, we have the opposite result. Therefore the skewed Student density distribution allows for asymmetric VaR forecasts and fully takes into

<sup>14</sup>These results are in line with those reported in Table 1 (skewed Student APARCH on daily returns).

<sup>15</sup>The quantile function of the (standardized) skewed Student has been derived in Lambert and Laurent (2001) as a mixture of two Student quantile functions. See also Giot and Laurent (2001b).

account the fact that the density distribution of asset returns can be substantially skewed. See for example Chen, Hong, and Stein (2001) for a discussion and motivation of skewness in financial returns.

All models are tested with a VaR level  $\alpha$  which ranges from 5% to 0.25% and their performance is then assessed by computing the failure rate for the returns  $y_t$ . By definition, the failure rate is the number of times returns exceed (in absolute value) the forecasted VaR. This measure is also called the proportion of VaR violations, where a VaR violation is defined as an occurrence of a market returns larger (in absolute value) than the forecasted VaR. If the VaR model is correctly specified, the failure rate should be equal to the pre-specified VaR level, i.e.  $\alpha$ . In our empirical application, we define a failure rate  $f_l$  for the long trading positions, which is equal to the percentage of negative returns smaller than one-step-ahead VaR for long positions (left tail of the density distribution of the returns), i.e.  $\frac{1}{T} \sum_{t=1}^T I(y_t < VaR_t(\alpha))$ , where  $I(\cdot)$  is the indicator function and  $VaR_t(\alpha)$  is the VaR forecast at time  $t$  and at  $\alpha$  %. Correspondingly, we define  $f_s$  as the failure rate for short trading positions as the percentage of positive returns larger than the one-step-ahead VaR for short positions (right tail of the density distribution of the returns). Using a procedure that is now standard in the VaR literature, we assess the models' performance by first computing their empirical failure rates, both for the left and right tails of the distribution of returns. Because the computation of the empirical failure rate defines a sequence of yes (VaR violation)/no (no VaR violation) observations, it is possible to test  $H_0 : f = \alpha$  against  $H_1 : f \neq \alpha$ , where  $f$  is the failure rate (estimated by  $\hat{f}$ , the empirical failure rate). In the literature on VaR models, this test is also called the Kupiec LR test, if the hypothesis is tested using a likelihood ratio test (see Kupiec, 1995). The LR statistic is  $LR = -2 \ln(\alpha^{T-N}(1-\alpha)^N) + 2 \ln\left(\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N\right)$ , where  $N$  is the number of VaR violations,  $T$  is the total number of observations and  $f$  is the theoretical failure rate. Under the null hypothesis that  $f$  is the true failure rate, the  $LR$  test statistic is asymptotically distributed as a  $\chi^2(1)$ .

Besides the failure rate, a relevant VaR model should feature a sequence of indicator functions (VaR violations) that is not serially correlated. With the new variables  $Hit_t(\alpha) = I(y_t < VaR_t(\alpha)) - \alpha$  and  $Hit_t(1-\alpha) = I(y_t > VaR_t(1-\alpha)) - \alpha$ , Engle and Manganelli (1999) suggest to test jointly that:

- A1:  $E(Hit_t(\alpha)) = 0$  (respectively  $E(Hit_t(1-\alpha)) = 0$ ) in the case of long trading positions (short trading positions);
- A2:  $Hit_t(\alpha)$  (or  $Hit_t(1-\alpha)$ ) is uncorrelated with the variables included in the information set.

According to Engle and Manganelli (1999), testing A1 – A2 can be done using the artificial regression  $Hit_t = X\lambda + \epsilon_t$ , where  $X$  is a  $T \times k$  matrix whose first column is a column of ones,

the next  $q$  columns are  $Hit_{t-1}, \dots, Hit_{t-q}$  and the  $k - q - 1$  remaining columns are additional independent variables (including the VaR itself). Engle and Manganelli (1999) also show that under the null  $A1 - A2$ , the Dynamic Quantile test statistic  $\frac{\hat{\lambda}' X' X \hat{\lambda}}{\alpha(1-\alpha)} \stackrel{a}{\sim} \chi^2(k)$ , where  $\hat{\lambda}$  is the OLS estimates of  $\lambda$ . A small sample version of this test (F-test) is readily obtained but the difference is negligible since the sample size is larger than 1,000 observations. Note that while Engle and Manganelli (1999) only consider long trading positions, we also use this test when computing the VaR of short trading positions.

Table 3 reports the P-values for the Kupiec (1995) failure rate test, while Table 4 presents the results for the Engle and Manganelli (1999) regression quantile tests with  $q = 5$  and  $k = 7$  (i.e. we include the contemporaneous VaR forecast as additional explanatory variable). For both tests, each panel presents successively the results for the RiskMetrics, SKST APARCH, normal RV and SKST RV models. Let us first compare the results given in rows 3 and 4 of each panel, i.e. the results for the normal and skewed Student realized volatility specification. The empirical results given in both tables tell the same story. First and for the normal RV specification, the P-values for the null hypothesis of both tests are often smaller than 0.05, especially when  $\alpha$  is below 1%. Secondly the skewed Student RV model performs very well as there are almost no P-values smaller than 0.05, whatever the test and the tail one takes into account. Thus the switch from the normal distribution to the skewed Student distribution yields a significant improvement in the VaR performance of the model set in the realized volatility framework. In a second step, we now compare the results given in rows 2 and 4 of each panel, i.e. the results for the skewed Student APARCH model (daily data only) and the skewed Student realized volatility specification (which uses intradaily data). It is quite striking that we get very close (from a qualitative point of view) results: in most cases, P-values are larger than 0.05, both for the model that uses the daily data and for the model that uses the intradaily data (realized volatility). Therefore, using an APARCH model with daily data or a two-step approach relying on the realized volatility leads to very similar results in terms of VaR performance, provided that one correctly specify the full conditional density (the skewed Student density distribution) for *both* methods.

This implies that previous results given in the empirical literature must be qualified. For example, Ebens (1999) concludes his paper by stating that the GARCH model underperforms (when volatility must be forecasted) with respect to the model based on the daily realized volatility. However, the author uses a GARCH model that neither really accounts for the long memory property observed in the realized volatility nor the fat-tails or asymmetry of the returns (even after standardization). Indeed, when estimating the more simple RiskMetrics VaR model on daily returns (the RiskMetrics model is tantamount to an IGARCH model with pre-specified coefficients, under the additional assumption of normality), we have the VaR results given in the first row of each panel of Table 3 and 4: its one-day-ahead forecasting performance is rather poor, especially

Table 3: VaR failure rate results for the stock indexes and exchange rates

$\alpha$	VaR for long positions					VaR for short positions				
	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
CAC40 stock index										
RiskMetrics	0.051	0.005	0.002	0.000	0.000	0.656	0.555	0.667	0.299	0.148
SKST APARCH	0.754	0.233	0.223	0.923	0.945	0.855	0.555	0.891	0.606	0.633
Normal RV	0.399	0.090	0.027	0.018	0.021	0.754	0.245	0.891	0.765	0.945
SKST RV	0.555	0.617	0.337	0.765	0.497	0.643	0.332	0.891	0.923	0.497
SP500 stock index										
RiskMetrics	0.301	0.001	0.000	0.000	0.000	0.301	0.125	0.036	0.004	0.060
SKST APARCH	0.472	0.737	0.541	0.843	0.519	0.575	0.655	0.018	0.096	0.010
Normal RV	0.058	0.655	0.036	0.001	0.000	0.025	0.570	0.803	0.960	0.333
SKST RV	0.809	0.911	0.916	0.660	0.692	0.809	0.650	0.031	0.050	0.010
YEN-USD exchange rate										
RiskMetrics	0.049	0.537	0.023	0.004	0.001	0.126	0.000	0.000	0.000	0.000
SKST APARCH	0.817	0.762	0.500	0.079	0.532	0.521	0.399	0.105	0.108	0.419
Normal RV	0.006	0.134	0.074	0.108	0.040	0.521	0.003	0.000	0.000	0.000
SKST RV	0.921	0.512	0.627	0.542	0.532	0.521	0.339	0.198	0.863	0.809
DEM-USD exchange rate										
RiskMetrics	0.043	0.017	0.000	0.000	0.000	0.934	0.333	0.001	0.000	0.000
SKST APARCH	0.869	0.377	0.391	0.859	0.635	0.617	0.687	0.435	0.250	0.900
Normal RV	0.934	0.529	0.004	0.000	0.000	0.128	0.377	0.195	0.023	0.001
SKST RV	0.934	0.519	0.942	0.501	0.900	1.000	0.519	0.913	0.669	0.417

P-values for the null hypotheses  $f_l = \alpha$  (i.e. failure rate for the long trading positions is equal to  $\alpha$ , top of the table) and  $f_s = \alpha$  (i.e. failure rate for the short trading positions is equal to  $\alpha$ , bottom of the table).  $\alpha$  is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%. The RiskMetrics and skewed Student APARCH models are estimated on the daily returns (i.e. no use is made of the intraday returns) while the last two models are estimated using the two-step approach described in Section 3.2 and use the intraday returns.

Table 4: VaR quantile regression results for the stock indexes and exchange rates

$\alpha$	VaR for long positions					VaR for short positions				
	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
CAC40 stock index										
Riskmetrics	0.232	0.028	0.000	0.000	0.000	0.878	0.923	0.999	0.973	0.889
SKST APARCH	0.650	0.539	0.175	0.000	1.000	0.524	0.112	0.999	0.995	0.959
Normal RV	0.908	0.338	0.097	0.000	0.000	0.971	0.750	0.998	0.999	0.995
SKST RV	0.972	0.287	0.164	0.000	1.000	0.943	0.787	0.998	1.000	1.000
SP500 stock index										
Riskmetrics	0.002	0.000	0.000	0.000	0.000	0.021	0.078	0.248	0.048	0.218
SKST APARCH	0.101	0.018	0.780	0.152	0.999	0.051	0.293	0.252	0.924	0.615
Normal RV	0.173	0.147	0.068	0.004	0.002	0.195	0.718	0.980	0.996	0.988
SKST RV	0.156	0.145	0.896	0.984	1.000	0.222	0.704	0.690	0.856	0.678
YEN-USD exchange rate										
Riskmetrics	0.012	0.075	0.119	0.030	0.001	0.013	0.000	0.000	0.000	0.000
SKST APARCH	0.264	0.325	0.975	0.903	0.999	0.694	0.016	0.002	0.000	0.968
Normal RV	0.036	0.019	0.056	0.004	0.000	0.480	0.000	0.000	0.000	0.000
SKST RV	0.572	0.031	0.464	0.015	0.790	0.480	0.058	0.019	0.140	1.000
DEM-USD exchange rate										
Riskmetrics	0.009	0.000	0.000	0.000	0.000	0.051	0.019	0.000	0.000	0.000
SKST APARCH	0.959	0.776	0.892	0.994	0.999	0.389	0.000	0.011	0.007	1.000
Normal RV	0.662	0.721	0.008	0.000	0.000	0.258	0.593	0.129	0.002	0.000
SKST RV	0.671	0.866	0.635	0.887	1.000	0.638	0.269	0.533	0.130	0.996

P-values for the Dynamic Quantile test statistic  $\frac{\hat{\lambda}'X'X\hat{\lambda}}{\alpha(1-\alpha)} \stackrel{\mathcal{L}}{\sim} \chi^2(7)$ .  $\alpha$  is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%. The RiskMetrics and skewed Student APARCH models are estimated on the daily returns (i.e. no use is made of the intraday returns) while the last two models are estimated using the two-step approach described in Section 3.2 and use the intraday returns.

when  $\alpha$  is small.<sup>16</sup> With a more ‘sophisticated’ model on the other hand, VaR results are much better. Interestingly (and this confirms what we showed in Section 3.3 where we compared the statistical properties of the ex-ante standardized returns obtained with the normal and skewed Student distributions), the same conclusion is true for the more complex model based on the combination of intraday returns and realized volatility. Therefore this also shows that dealing with realized volatility in a forecasting framework does not mean that the normal distribution is a natural and obvious choice for the distribution of returns (which it is for ex-post standardized returns). In other words, a forecasting framework where realized volatility is used implies that this measure should be used in conjunction with an adequate density distribution.

## 5 Conclusion

In this paper we have shown how to compute a daily VaR measure for two stock indexes (the CAC40 and SP500 stock indexes) and two exchange rates vis-a-vis the US dollar (YEN-USD and DEM-USD) using volatility forecasts based on realized volatility. The daily realized volatility is equal to the sum of the squared intraday returns over a given day and thus uses intraday information to define an aggregated daily volatility measure. While the VaR forecasts which use this method perform adequately over our sample, we also show that a more simple model based solely on daily returns also delivers nice results. Indeed, while the VaR specification based on an ARFIMAX(0, $d$ ,1)-skewed Student model for the daily realized volatility provides adequate one-day-ahead VaR forecasts, it does not really improve on the performance of a VaR model based on the skewed Student APARCH model and estimated using daily data. Thus, for the four financial assets considered in our study, the two methods seem to be rather equivalent provided that one correctly specify the full conditional density (the skewed Student density distribution) for *both* methods and one does not use the normal distribution (even in the realized volatility framework). Another important conclusion of our paper is that daily returns standardized by the square root of the one-day-ahead forecast of the daily realized volatility are not normally distributed. This extends recent results on realized volatility (such as Andersen, Bollerslev, Diebold, and Ebens, 2001) and shows that, in a forecasting framework such as Value-at-Risk, realized volatility measures must be combined with adequate density distributions.

At this stage, one of the most immediate and promising extension of these techniques is to consider corresponding multivariate volatility models to forecast the VaR of a portfolio of financial assets. Multivariate models of the ARCH type are not easy to implement as they often require the estimation of a large number of parameters. Furthermore, these parameters are present in the

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<sup>16</sup>Although the results are not reported in the chapter, we also estimated a normal GARCH(1,1) model and its performance was not much better than the RiskMetrics specification.

latent volatility specification and this is one of the main difficulty of the problem. Therefore, multivariate realized volatility models should provide a much easier way to correctly model variances and correlations across financial assets as they assume that volatility is observed. This paves the way for the use of ‘usual’ multivariate models (VAR, ECM) directly applied to realized volatility and correlations.

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